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Maintenance Contract of a Revenue-Earning Asset with Discrete Preventive Maintenance

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Abstract: This paper deals with maintenance service contract for a dump truck sold with a one-dimensional warranty. We consider a situation where an agent offers two maintenance contract options and the owner of the equipment has to select the optimal option. The option are the OEM only carries out failure below a specified value while the costumer undertakes preventive maintenance action in-house and the OEM carried out all repairs and preventive maintenance activities.Moreover, we study the maintenance service contract considering reduction of the intensity function after preventive maintenance from both the owner and OEM point of views. We use a non-cooperative game formulation to determine the optimal price structure (i.e., price of each contract and maintenance effort) for the OEM and the optimal option for the owner. The result derived from the model has shown that if the owner choose option with maintenance contract then the owner obtain a higher profit compared with the profit resulted from in-house maintenance.

Keywords:maintenance contract, a non-cooperative game theory, preventive maintenance, availability,warranty.

1. Introduction

Many heavy equipment such as cranes, loaders and dump trucks are common used in a mining industry to support its business. The equipment deteriorates with usage and age and finally failure occurs it it does not operate as intended. If the equipment is down for repair or preventive maintenance (PM), no revenue is generated and hence a high availability of the equipment is needed for achieving the production target of a company. To achieve a high availability of the equipment, Preventive Maintenance (PM) actions are performed to reduce the likelihood of failure and down time. Corrective Maintenance (CM) actions are taken after failure, which restores the failed equipment to the operational state.

Recently most heavy equipment are sold with warranty and often the manufacturer of Original Equipment Manufacturer (OEM) offers the warranty and PM in one package in order to provide more assurance to the owner of the equipment. After the warranty expires, the owner is fully responsible to perform the maintenance actions (PM and CM actions), which can be done either in house or by independent agents or the OEM. For complex and expensive equipment used in a remote mining areas, it is very expensive for the owner to have maintenance facility and high skill maintenance crew required if the maintenance is done in house. Hence, performing PM and CM in house is not economical after the warranty ends. An alternative solution is contracted maintenance activities to an external agent.

Usually, the owner wants to reach high availability of the equipment with lower cost and the external agent gives a variety of maintenance service contract. This in turn will result in optimal profits for both the owner and the agent.

This paper is organized as follows. Section 2 gives literature review related to maintenance contract for repairable equipment and description of our problem is presented in Section 3. In Sections 4, we present the result and analytical solution. Finally, we conclude with topics for further research.

2. Literature Review

Maintenance service contracts have received much attention in the literature. Jackson and Pascual (2008) and Wang (2010) studied maintenance service contracts for repairable items, which involve preventive maintenance policies. Those papers studied maintenance service contract with consider a penalty based on down time for each failure – i.e. a penalty cost incurs the agent (or OEM) when the actual down time to fix the failed equipment is greater than the target value. Iskandar *et al*.(2014) have studied service contracts with availability as a key measure, where a penalty cost incurs when the actual availability falls below the target (or total down time in a given period exceeds the target). But the service contracts studied do not give any reward to the agent when the performance (actual availability) well above the target. As a result, the service contracts do not motivates the agent (or OEM) to keep improving the performance. In this paper we introduce a policy limit cost asreward and protection to a service provider (an agent) from over claim and to pursue the owner to do maintenance under specified cost in house.Here we also consider the penalty based on the availability per period (usually one year) –i.e. if it is lower than the target availability the OEM incurs the penalty cost.

3. Problem Statement

According to the literature review, as the OEM or an external agent normally offers a variety of maintenance service contracts, then the maintenance actions (PM and/or CM) can be outsourced to the OEM (or an external agent). From the owner's viewpoint, maintenance programs are aimed at not only to reach the performance target (e.g. 90% availability) but also to achieve an optimal profit. In order to reach the optimal profit, the maintenance service contract offered by the OEM should not just to ensure the performance target but also to achieve a higher performance which is beyond the target. This in turn will results in optimal profits for both the owner and the OEM. The decision problems for the owner are (i) to select the maintenance contract option that can reach the higher performance of the equipment with reasonable maintenance costs, and (ii) determine an attractive cost. And for the OEM or an external agentthe decision problem is to determine the optimal price for each options offered.

The maintenance service contracts studied is theone which offers policy limit cost to protecta service provider (an agent) from over claim and to pursue the owner to do maintenance under specified cost in house. This in turn gives benefit for both the owner of the trucks and the agent of service contract. The decision problem for an agent is to determine the optimal price for each options offered and for the owner is to select the best contract option. We use a Nash game theory formulation in order to obtain a win-win solution – i.e. the optimal price for the agent and the optimal option for the owner.

We define the following notation that will be used in model formulation.

W,U	:Warranty time, and usage limits
X	: Product age
L	: life cycle
$\tilde{A}$	: Availability target
ζ	: Total downtime target
$Y(t)$	: Total downtime in $(0,t]$
$EP(t)$	: Expected penalty cost
$F(x)$	: Distribution function of downtime
$F^{[k]}(x)$	: The *k-fold Stieltjes convolution*of *F(x).*
$r(x)$	: Hazard function
$R(x)$	: Cumulative hazard function
$F(x),\ f(x)$	: Distribution function, density function X
α	: Scale parameter.
β	: Shape parameter.
P_G	: Service contract cost
P_0	: PM cost done by owner
K	:Revenue
C_m	:Repair cost done by OEM

C_s	: Repair cost option O_0 owner
C_{pm}	: Preventive maintenance cost per unit time.
C_p	: Penalty cost
C_b	: Price of the product.

3.1. Equipment Failures and Repairs

We use a black-box approach to model equipment failure. Every failure is fixed by a minimal repair or the failure rate after repair is the same as that before it fails. It is assumed that the repair time is very small compared to its mean time between failures, hence it can be ignored. As a result, the failure occurs as a Non-Homogenous Poisson process (NHPP) with the intensity function (Barlow and Hunter, 1960). To keep the equipment in good condition, PM is conducted regularly. PM can be done in-house or by the OEM or an agent. We consider that PM done in-house is less effective than that of the OEM. We model the effect of PM through the failure rate function as follows. If represents the failure rate function for the equipment with PM done in-house, then it is given by

$$r_0(x) = \begin{cases} r(x) & 0 \le x < W \\ r(W) + \eta r(x) & W \le x < L \end{cases} \quad (1)$$

where $\eta > 1$.

We consider that PM done by the OEM is an imperfect PM policy.The PM policy is characterised by single parameter $\tau[\nu]$ during Ω_W [Ω_S].The equipment is periodically maintained at $k.\tau$ [$l.\nu$]. Any failure occurring between pm is minimally repaired (See Fig. 1). Here the warranty ceases at W.The effect of imperfect PM actions on the intensity function is given by $r(t_j) = r(t_{j-1}) - \delta_j$ with $0 \le \delta_j \le r(t_{j-1}) - \sum_{i=0}^{j} \delta_i$, δ_j denotes the reduction of the intensity function after j^{th}, $j \ge 1$, PM action.If the PM action is done at j^{th}, $j \ge 1$ theintensity function is reduced by δ_j , then for $t_j \le t < t_{j+1}$ the intensity function is given by $r_j(t) = r(t) - \sum_{i=0}^{j} \delta_i$ with $\delta_0 = 0$. For simplicity we assume that for each PM action $\delta_j = \delta_{j+1} = \delta$ then $r_j(t) = r(t) - j\delta$.

If any failure occurring between pm is minimally repaired, then the expected total number of minimal repairs in $([t_{j-1}, t_j), 1 \le j \le k+1)$ is givenby

$$N = \sum_{j=1}^{k+1} \int_{t_{j-1}}^{t_j} r_{j-1}(t')dt' = R(W) - \sum_{j=1}^{k} \left(W - j\tau\right)\delta_j \ .$$

For $t_j - t_{j-1} = \tau_y$ then the expected number of minimal repairsin $[0,W)$ is defined as

$$N(W) = N(k,\tau) = R(0,W) - \sum_{j=1}^{k} \left[(W - j\tau)\right]\left[r(j\tau) - r((j-1)\tau)\right] \quad (2)$$

where $R_0(0,W) = \int_0^W r_0(t)dt - \sum_{j=1}^{k} \delta_j$.

And after the warranty ends, the expected number of minimal repairs in [W, W+L), with $1 \le m \le \ell + 1$ is given by

$$N(L) = N(\ell,\tau) = R_k(W,W+L) - \sum_{m=1}^{l} (L - j\nu)\delta_m = R_k(W,W+L) - \sum_{m=1}^{\ell} \left[(L - i\nu)\right]\left[r(i\nu) - r((i-1)\nu)\right] \quad (3)$$

where $R_k(W,W+L) = \int_W^L r_0(t)dt - \sum_{m=1}^{\ell} \delta_m$.

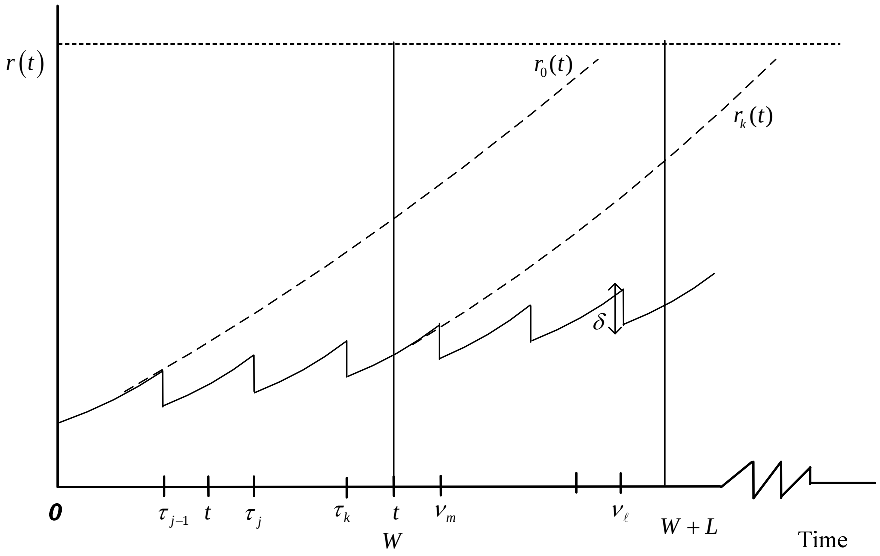

Figure 1. Failure Rate Function

For PM done by OEM, the failure rate function is given by

$$r_1(x) = r(x) \quad 0 \le x < L \tag{4}$$

Note that $\eta > 1$ meaning that the failurerate function increases with a higher rate or the PM done in-house is less effective than PM by OEM ($\eta = 1$).

3.2. Maintenance Service Contract

OEM's Decision Problem

As mentioned in the earlier section, the equipment under consideration is sold with warranty and the warranty also covers PM. The manufacturer will rectify all failures and preventive actions during the warranty period without any charge to the consumer. The consumer will be responsible for all CM and PM actions, after the warranty ceases. Hence, a comprehensive maintenance program of the equipment over a life cycle, L, is needed by the company to give a maximum availability of the equipment.

We consider that an agent offers two options to the customer after the warranty ends –ie. Option O_0 and O_1.Two service contract options are considered as follows. Option O_0: After the expiry of warranty or in , the consumer carries PM in-house. If the equipment fails, the owner calls the OEM to fix the equipment. The OEM will charge the consumer for the full cost of each repair. There is no penalty cost to the OEM if the availability falls below the target. The OEM will charge the higher cost of repair under this option. Option O_1 : For a fixed price of service contract P_G, the OEM agrees to carry out PM and CMin $[W, L)$ .The owner is required to pay an additionalcost if the repair cost is greater than a threshold value ϑ.If the availability falls below the target, the OEM should pay a penalty cost.Here, the OEM performs PM and CM over the life cycle of the equipment.

Generally, several packages of maintenance service contract offers by the OEM – e.g. full coverage or partial coverage. Here PM is full coverage.Under Option O_1 , the OEM provides a service covering PM and CM to theconsumer with a fixed cost P_G within the contract period –e.g. 3 to 5 years. If the availability of the equipment for period j, A_j is less thanthe availability target $\tilde{A}_j$, then the OEM should pay a penaltycost. The penalty costisproportional to $\delta_j = \tilde{A}_j - A_j$. The penalty cost, C_p is viewed as acompensation given by the OEM.The OEM needs to determine the optimal price structure (i.e. service contract cost for option and repaircost for option) to maximize the expected profit. Let *C* denote the repair cost to fix the failed equipment. As in many cases, the repair costvaries, then *C*is considered as a random variable with distribution function *G(c)*.Since every failure is fixed by a minimal repair then the failure process follows the Non-Homogeneous Poisson Process [9], and hence the expected of OEM revenue for option O_0 is given by

$$E\left[\pi\left(O_0\right)\right]=R_0\left(W,L\right)\left[\int_0^{\infty} cg(c)dc\right] \quad (5)$$

where $R_0\left(W,L\right)=\int_W^L r_0(x)dx$.

For option 1, first we obtain the expected of repair cost, expected of penalty cost and then expected of PM cost in (W, L) .

Under Option 1, if repair cost, C is greater than ϑ then the owner has to pay $(C-\vartheta)$. As a result the expected additional repair cost paid by the owner is given by $\int_{\vartheta}^{\infty}(c-\vartheta)g(c)dc$. Then, the expected of repair cost incurred by the OEM is

$$EC_m = R_1\left(W,L\right)pC_m \quad (6)$$

Where $C_m=\int_0^{\infty} cg(c)dc$ and $R_1\left(W,L\right)=\int_W^L r(x)dx$.

Expected of Penalty Cost:

Let $Y(t)$ and $A(t)$ denote the total of down time and availability of the equipment in *(0,t)*. If $\tilde{A}$ is the availability target, then a penalty occurs if $A(t)<\tilde{A}$ (or $1-Y(t)/t<\tilde{A}$). If $\zeta=(1-\tilde{A})t$, then the penalty incurs the OEM if $Y(t)>\zeta$ or the total down time in *(0,t)* is greater than ζ . The expected penalty cost is given by $EP(t)=c_p E[Max\{0,Y(t)-\zeta\}]/t$ where c_p is the penalty cost and

$$E\left[Max\{0,Y(t)-\zeta\}\right]=\int_{\zeta}^{\infty}\left(y-\zeta\right)g(y)dy \quad (7)$$

Hence, we have

$$EP(t)=\frac{c_p\left\{\int_{\zeta}^{\infty}(y-\zeta)g(y)dy\right\}}{t} \quad (8)$$

In most cases, availability target is given for each year. As the equipment deteriorates with age and usage, the availability target decreases from year to year. Let $\tilde{A}_j,\ j=1,\ldots,(L-W)$ denote the availability target at t_j, then $\tilde{A}_1<\tilde{A}_2<\ldots<\tilde{A}_{L-W}$.

Let $Y(t_{j-1},t_j)$ and $A(t_{j-1},t_j)$ denote the total of down time and availability of the equipment in (t_{j-1},t_j), then $A\left(t_{j-1},t_j\right)=1-Y\left(t_{j-1},t_j\right)/\left(t_j-t_{j-1}\right)$. The penalty incurs the OEM at time t_j if $A\left(t_{j-1},t_j\right)<\tilde{A}_j$ or $Y\left(t_{j-1},t_j\right)>\zeta_j$ (the total down time in (t_{j-1},t_j) is greater than ζ_j), where $\zeta_j=\left(1-\tilde{A}_j\right)\left(t_j-t_{j-1}\right)$.

Hence, the probability thatthe penalty incurs at t_j is given by

$$\mathrm{P}\left\{Y(t_{j-1},t_j)>\zeta_j\right\}=\sum_{k=1}^{\infty}\left\{1-F^{[k]}\left(\zeta_j\right)\right\}\frac{R(t_{j-1},t_j)^k e^{-R(t_{j-1},t_j)}}{k!} \quad (9)$$

Define,

$$G_j(\zeta_j)=\mathrm{P}\left\{Y\left(t_{j-1},t_j\right)\leq\zeta_j\right\}$$

From (9), we have

$$G_j(\zeta_j) = \sum_{k=1}^{\infty} F^{[k]}\left(\zeta_j\right) \frac{R(t_{j-1},t_j)^k e^{-R(t_{j-1},t_j)}}{k!} \quad (10)$$

We assume that $F(x)$ has Exponential distribution with parameter λ , then we have after simplification,

$$G_j(\zeta_j) = \mathrm{P}\left\{Y(t_{j-1},t_j) \le \zeta_j\right\} = \sum_{k=1}^{\infty} \frac{R(t_{j-1},t_j)^k e^{-R(t_{j-1},t_j)}}{k!} \frac{(\lambda\zeta_j)^k}{k!} e^{-\lambda\zeta_j} \quad (11)$$

And the density function of $G_j\left(\zeta_j\right)$ is given by $g_j(\zeta_j) = \dfrac{dG_j(\zeta_j)}{d\zeta_j}$.

Now, we obtain the expected penalty cost in (t_{j-1},t_j) and then in (W,L). The expected penalty cost in (t_{j-1},t_j) is given by

$$EP_j\left(t_{j-1},t_j\right) = \frac{c_p E\left[Max\left\{0, Y\left(t_{j-1},t_j\right) - \zeta_j\right)\right\}\right]}{\left(t_j - t_{j-1}\right)} = \frac{c_p \int_{\zeta_j}^{\infty} \left[1 - G_j\left(y\right)\right] dy}{\left(t_j - t_{j-1}\right)} \quad (12)$$

As a result, the expected penalty cost in (W,L) is given by

$$EP(W,L) = \begin{cases} \displaystyle\sum_{j=1}^{L-W} \frac{c_p \int_{\zeta_j}^{\infty} \left[1 - G_j\left(y\right)\right] dy}{\left(t_j - t_{j-1}\right)} & \text{for} \quad A_j < \tilde{A}_{\mathrm{j}} \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

The expected of PM cost is

$$EC_{pm} = k_y C_0 - \sum_{j=1}^{k_y+1} \left[C_r\left(L - j\tau_y\right) - C_v\right]\left[r\left(j\tau_y\right) - r\left(\left(j-1\right)\tau_y\right)\right] \quad (14)$$

As a result, the total expected revenue of the OEM is

$$E\left[\pi(O_1)\right] = P_G - C_p \sum_{j=1}^{L-W} \frac{\int_{\zeta_j}^{\infty} \left[1 - G_j\left(y\right)\right] dy}{\left(t_j - t_{j-1}\right)} - R_1\left(W,L\right) C_m - EC_{pm} \quad (15)$$

<u>Owner's Decision Problem</u>

The owner needs to decide which options best fit to maintain the equipment over *(W,L)* – i.e. to decide whether a PM is done in house and CM by the OEM or both PM and CM is fully done by the OEM. As a result, after the expiry of warranty, the consumer must choose the option O^* taken from the set $\left\{O_0, O_1\right\}$. As theequipment isused to generate income, then the owner has to select the optimal option that maximizes the expected profit.

The expected profit of the consumer upon choosing the O_0 option, $E\left[\omega(O_0)\right]$ is given by

$$\begin{aligned} E\left[\omega(O_0; P_0, \vartheta)\right] = {} & K\left[L - W - \left(R_0(L-W) - R_0(W)\right) E\left[U_i\right]\right] \\ & - R_0\left(W,L\right)\left(C_m - C_s\right) - P_0 - C_b \end{aligned} \quad (16)$$

While the expected profit for Option O_1 is given by

$$\begin{aligned} E\left[\omega\left(O_1\right)\right] = {} & K\left[L - W - \left(R_1(L-W) - R_1(W)\right) E\left[U_i\right]\right] + C_p \sum_{j=1}^{L-W} \frac{\int_{\zeta_j}^{\infty} \left[1 - G_j\left(y\right)\right] dy}{\left(t_j - t_{j-1}\right)} \quad (17) \\ & - R_1\left(W,L\right) \int_{\vartheta}^{\infty} \left(c - \vartheta\right) g(c) dc - P_G - C_b \end{aligned}$$

4. Result

We first obtain P_G^* and then C_s^*. In thepresence of negotiation between the two parties for every optionoffered, the consumer and the OEM will receive the same profit. Then, for Option O_1 we have P_G^* given by

$$P_G^* = \frac{1}{2}\left[\begin{array}{l} K\left[L - W - \sum_{j=1}^{L-W} E\left[Y\left(t_{j-1}, t_j\right)\right]\right] + 2C_p \sum_{j=1}^{L-W} \frac{\int_{\zeta_j}^{\infty}\left[1 - G_j\left(y\right)\right]dy}{\left(t_j - t_{j-1}\right)} \\ + R_1\left(W, L\right)\left(C_m - \int_{\vartheta}^{\infty}(c-\vartheta)g(c)dc\right) + EC_{pm} - P_G - C_b \end{array}\right] \tag{18}$$

$$\sum_{j=1}^{L-W} E\left[Y\left(t_{j-1}, t_j\right)\right] = \left(R_1(L-W) - R_1(W)\right)E\left[U_i\right]$$

The expected profit of the OEM on option O_1 , becomes

$$E\left[\pi(O_1; P_G)\right] = \frac{1}{2}\left[\begin{array}{l} K\left[L - W - \sum_{j=1}^{L-W} E\left[Y\left(t_{j-1}, t_j\right)\right]\right] - R_1\left(W, L\right)\left(C_m - \int_{\vartheta}^{\infty}(c-\vartheta)g(c)dc\right) \\ -EC_{pm} - C_b \end{array}\right] \tag{19}$$

We have C_s^* for Option O_0 that satisfied

$$C_s^* = \frac{1}{2R_0\left(W, L\right)}\left[K\left\{L - W - \sum_{j=1}^{L-W} E\left[Y\left(t_{j-1}, t_j\right)\right]\right\} + C_m R_0\left(W, L\right) - P_0 - C_b\right] \tag{20}$$

$$E\left[\pi(O_0; P_0, C_s)\right] = \frac{1}{2}\left[K\left\{L - W - \sum_{j=1}^{L-W} E\left[Y\left(t_{j-1}, t_j\right)\right]\right\} - P_0 - C_b - C_m R_0\left(W, L\right)\right] \tag{21}$$

Here, we can see that maximum expected profit for the OEM using option O_1 is always greater than using option O_0. It can be seen by the difference of (19) and (21), i.e,

$$E\left[\pi(O_1; P_G)\right] - E\left[\pi(O_0; C_s)\right] = \frac{1}{2}\left[P_0 + C_m R_0\left(W, L\right) - R_1\left(W, L\right)\left(C_m - \int_{\vartheta}^{\infty}(c-\vartheta)g(c)dc\right) - EC_{pm}\right] > 0$$

since all parameter is positive, $P_0 > EC_{pm}$, and $R_0\left(W, L\right) > R_1\left(W, L\right)$.

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